The Gyárfás’ path argument and algorithms
Marcin Pilipczuk

Graph classes excluding a path or a subdivided claw as an induced subgraph are so far quite mysterious: on one hand, beside some corner cases, they do not seem to have any good structural description, but on the other hand, a number of combinatorial problems - including Maximum Independent Set (MIS) - lack an NP-hardness proof in these graph classes, indicating a possible hidden structure that can be used algorithmically. Furthermore, graphs excluding a fixed path as an induced subgraph were one of the earliest examples of a chi-bounded graph class, with an elegant proof technique dubbed the Gyárfás’ path argument. In the recent years the progress accelerated, leading to, among other results, (a) a quasi-polynomial-time algorithm for MIS in graphs excluding a fixed path as an induced subgraph, (b) a QPTAS for MIS in graphs excluding a subdivided claw as an induced subgraph, (c) the proof of the Erdős-Hajnal property in graph classes excluding a fixed forest and its complement. In the talk, I will survey some of these results, showing the role of the Gyárfás’ path argument in most (all?) of them, and outline research directions for the future.